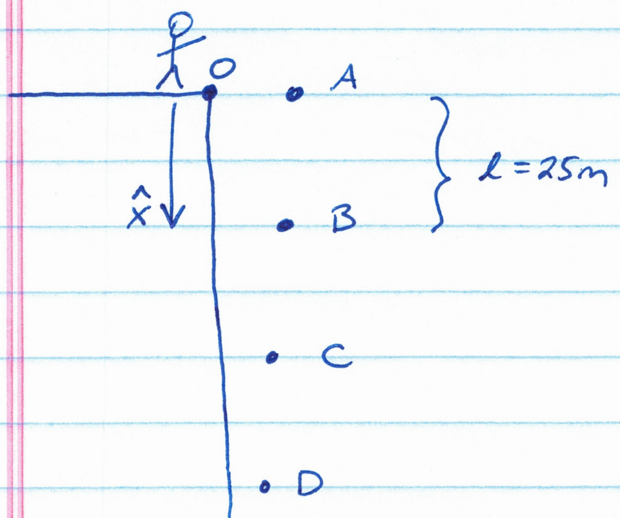
A bungee cord attached to a tower behaves like a spring of constant $k = 29\ N/m$ and unstretched length $\ell = 25\,m$. Miyako, whose mass is $m = 55\,kg$, steps off the tower.

a) How far does she fall?

b) What is her max acceleration?



We choose A to be the reference level for gravitational potential $\mathcal{E}$. So $U_{grav}(A) = 0$. (*)

Between A and B she is in free fall. Past B, the bungee begins to stretch.

(*) It should be pointed out that her <u>total energy</u> is zero. All Ke and PE is zero at point A, and zero it must remain!!

We expect her to oscillate between B and D like a mass on a spring, with $\omega = \sqrt{\frac{k}{m}}$

At point D she is momentarily at rest before being pulled upwards. So x_D is he max fall distance.

Let's write down her potential $\mathcal{E}$ as a function of x. Recall that $PE = PE(x) = U_{grav}(x) + U_{spring}(x)$.

$$U(x) = \begin{cases} -mgx & x < \ell \\ -mgx + \frac{1}{2}k(x-\ell)^2 & x > \ell \end{cases}$$

We know she oscillates between X_B and X_D. Her equilibrium point is (obviously) midway between these points, at X_C.

We also know that energy is going back and forth between kinetic and potential. Since total $\gamma = Ke(x) + PE(x)$ is conserved, her PE is minimized at X_C since her Ke is maximized at X_C; she is going the fastest at the midpoint of her oscillations.

$$\Rightarrow \text{PE is minimized at } X_C \Rightarrow \frac{dU}{dx}(X_C) = 0$$

so let's use this fact to get X_C:

$$\frac{dU}{dx} = \frac{d}{dx}\left[-mgx + \frac{1}{2}k(x-\ell)^2\right]$$

$$X_C = -mg + k(X_C - \ell) \equiv 0$$

$$\text{so, } X_C = \ell + \frac{mg}{k} = 43.6\,m$$

Next, her Ke and PE are both 0 at X_A, so her total energy is zero.

Furthermore, at X_D her Ke is zero, so her PE also has to be zero:

$$-mgX_D + \frac{1}{2}k(\ell - X_D)^2 = 0$$

$$U(x_D) = \tfrac{1}{2}kx_D^2 - k\ell x_D + \tfrac{1}{2}k\ell^2 - mgx_D$$

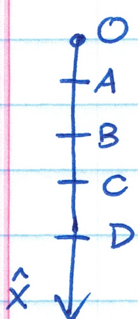
$$= x_D^2 - 2x_D\left(\ell + \tfrac{mg}{k}\right) + \ell^2$$

Note that $(\ell + \frac{mg}{k}) = x_c$. Since $U(x_D) = 0$, we have ourselves a nice quadratic eqn in x_D:

$$x_D^2 - 2x_D x_c + \ell^2 \equiv 0$$

$$\Longrightarrow \quad x_D = x_c \pm \sqrt{x_c^2 - \ell^2}$$

Since we know x_c is closer to the origin than x_D, we need the root where x_D is <u>larger</u> than x_c: the positive root.



$$\boxed{x_D = x_c + \sqrt{x_c^2 - \ell^2}}$$

OK, that was a little long. Lets talk about acceleration. We know that from:

$$|a(t)| = |\ddot{x}(t)| = \omega^2 A\cos(\omega t + \phi)$$

we need to find ω and A to compute the maximum acceleration.

For ω we just need to remember ω for a mass on spring:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{29.0\,N/m}{55.0\,kg}} = 0.726\ rad/s$$

What about A? She oscillates between X_B and X_D so A will be half that distance:

$$A = X_D - X_C$$

$$= \sqrt{X_C^2 - \ell^2}$$

$$= 35.7\,m$$ ← wow!!

So,

$$|a_{max}| = \omega^2 A$$
$$= (0.726\,rad/s)^2 \cdot 35.7\,m$$
$$= 18.8\ m/s^2$$

The problem didn't ask for this, but $18.8\ m/s^2 \approx$ "2g".

How does this compare with what we normally experience? A typical roller coaster is ~4g... twice as much. So in comparison with roller coasters, bungee jumping is quite tame. The "scare" comes from "the visuals" of tossing yourself off a cliff. The actual motion itself is not that bad. The strongest roller coasters reach 6g. Fighter pilots often experience 8g, but mostly in training.