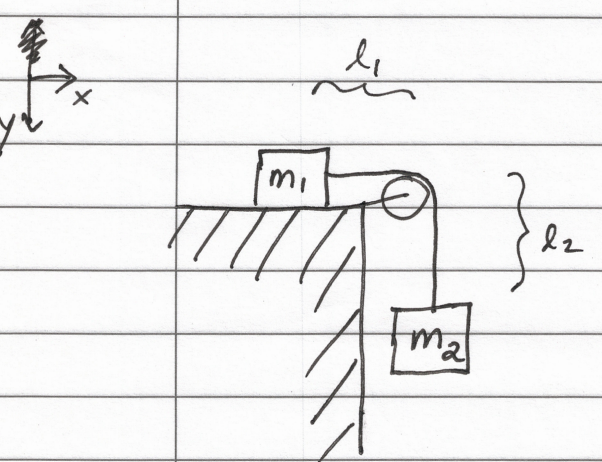
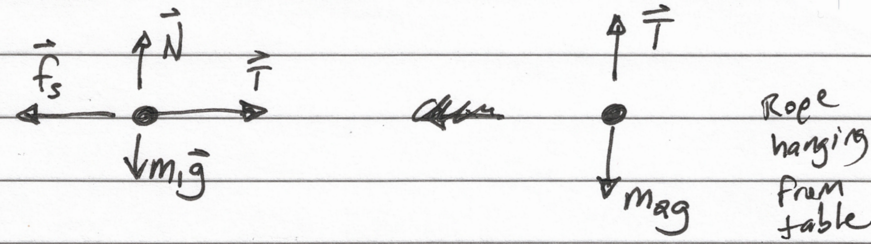
Two masses, m_1 and m_2, are configured as shown. The coefficient of static friction between m_1 and the surface is μ.

What relationship between m_1 and m_2 exists such that m_1 just barely remains stationary?

Note: This appeared on previous AP tests!





Here, $m_1 + m_2 = M$ is the mass of the entire string. Let λ denote the ratio of the length of string still on the table, so:

$$m_1 = \lambda M \qquad \text{(i)}$$

$$m_2 = (1-\lambda)M \qquad \text{(ii)}$$

Here we're assuming that the rope is of uniform density so $m_1 \propto l_1$ and $m_2 \propto l_2$ and $M \propto L$. Clearly, f_s, N, and T are all functions of λ. Developing our equations:

$$N - \lambda M = m a_{1y} = 0 \qquad \text{(iii)}$$

$$T - f_s = m a_{1x} = 0 \qquad \text{(iv)}$$

$$T - (1-\lambda)M = m a_{2y} = 0 \qquad \text{(v)}$$

And lastly, static friction can vary between $[0, \mu_s N]$. However, we're considering the case where the rope is *just* about to slide off the table, so:

$$f_s \equiv f_{s,max} = \mu_s N \qquad \text{(vi)}$$

I think (?) what we want is the proportion of the rope on the table to the total rope length such that the rope just barely stays stationary. This proportion is:

$$\frac{m_1}{M} = \frac{\lambda M}{M} = \lambda$$

So we want to find λ in terms of known quantities. Just to be clear – λ is the proportion of the string on the table to the total length of string.

At this point, it's nothing more than an algebra problem. Starting with (iii):

$$\lambda = \frac{N}{M} = \frac{1}{M}\left(\frac{f_{s,max}}{\mu_s}\right) = \frac{1}{M}\frac{T}{\mu_s}$$

Using (iv):

$$\lambda = \frac{1}{\cancel{M}}\frac{(1-\lambda)\cancel{M}}{\mu_s} \qquad \leftarrow \text{Interesting cancellation}$$

Solving for λ,

$$\boxed{\lambda = \frac{1}{\mu_s + 1}}$$

Sort of an interesting answer, no? Does it make sense? We expect λ to be a number on the interval $[0, 1]$, right? If $\lambda = 0$ the rope is all dangling. If $\lambda = 1$ the rope is all on the table. Observe:

$$\left.\begin{array}{l} \bullet\ \lambda(\mu_s = 0) = 1 \\ \bullet\ \lambda(\mu_s \to \infty) \to 0 \end{array}\right\} \text{check!!}$$

Also note that if the table is very slippery, μ_s is a low number, causing λ to approach 1, meaning that for slippery surfaces, most of the rope has to lie on the table. Conversely, for high friction, μ_s is very large, causing λ to approach 0, meaning most of the rope dangles. Totally makes sense! ☺ It <u>is</u> interesting that the answer does not depend on the rope's mass <u>at</u> <u>all</u>.